Currency Asian Option

Currency Asian

An Currency Asian option or Asian currency option is a special type of option contract where the payoff depends on the average of the underlying foreign exchange rate over a certain period of time. The payoff is different from the case of a European option or American option, where the payoff of the option contract depends on the underlying FX rate at exercise date.

Options give market participants many opportunities to limit risk and increase profit. Currency market fluctuations can have a lasting impact on cash flow whether it is buying a property, paying salaries, making an investment or settling invoices. By utilizing Currency Asian Options, business can protect themselves against adverse movements in exchange rates.

Asian Option Introduction

- An Asian option or *average* option is a special type of option contract where the payoff depends on the average of the underlying exchange rates over a certain period of time
- Asian options allow the buyer to purchase or sell the underlying asset at the average rate instead of the spot rate.
- Average rate options are less expensive than regular options and are arguably more appropriate than regular options for meeting some of investment needs.
- The payoff is different from the case of a European option or American option, where the payoff of the option contract depends on the rate of the underlying asset at exercise date.

Asian Option Introduction (Cont.)

- One advantage of Asian options is that they reduce the risk of market manipulation of the underlying instrument at maturity.
- Another advantage of Asian options involves the relative cost of Asian options compared to European or American options. Because of the averaging feature, Asian options reduce the volatility inherent in the option; therefore, Asian options are typically cheaper than European or American options.
- Asian options have relatively low volatility due to the averaging mechanism. They are used by traders who are exposed to the underlying asset over a period of time.

Asian Option Introduction (Cont)

- The arithmetic average rate options are generally used to smooth out the impact from high volatility periods or prevent rate manipulation near the maturity date, which makes the options less expensive.
- Currency options are one of the most common ways for corporations, individuals or financial institutions to hedge against adverse movements in exchange rates.
- Corporations primarily use FX options to hedge uncertain future cash flows in a foreign currency. The general rule is to hedge certain foreign currency cash flows with forwards, and uncertain foreign cash flows with options.

Forex Market Convention

- One of the biggest sources of confusion for those new to the FX market is the market convention. We need to make clear the meaning of the following terms in the forex market first.
- **FX quotation**: the quotation EUR/USD 1.25 means that one Euro is exchanged for 1.25 USD. Here EUR (nominator) is the base or primary currency and USD (denominator) is the quote currency. One can convert any amount of base currency to quote currency by

  QuoteCurrencyAmount = FxRate * BaseCurrencyAmount

Forex Market Convention (Cont.)

- **Spot Days**: The spot date or value date is the day the two parties actually exchange the two currencies. In other words, a currency pair requires a specification of the number of days between the quotation date (trade date) and the Spot Date on which the exchange is to take place at that quote. Spot days can be different for each currency pair, although typically it is two business days.
- **Holidays**: Each currency pair has a set of holidays associated with it. The holidays of a currency pair is the union of the holidays of the two currencies.

Valuation

- The payoff of an average rate call is max(0, X_{avg} - K) and that of an average price put is max(0, K- X_{avg}), where X_{avg} is the average rate of the underlying asset calculated over a predetermined averaging period.
- If the underlying exchange rate, X, is assumed to be lognormally distributed and X_{avg} is a geometric average of the X'*s*, analytic formulas are available for valuing European average rate options. This is because the geometric average of a set of lognormally distributed variables is also lognormal.
- When, as is nearly always the case, Asian options are defined in terms of arithmetic averages, exact analytic pricing formulas are not available. This is because the distribution of the arithmetic average of a set of lognormal distributions does not have analytically tractable properties.

Valuation (Cont.)

- However, the distribution of arithmetic average can be approximated to be lognormal by moment matching technical, which leads to a good analytic approximation for valuing average price options.
- One calculates the first three moments of the probability distribution of the arithmetic average in a risk-neutral world exactly and then fit a lognormal distribution to the moments.

$$M_1 = \sum_{i=0}^{n-1} \beta_i$$

$$M_2 = \sum_{i=0}^{n-1} \beta_i e_i \left(2 \sum_{j=i}^{n-1} \beta_j - \beta_i\right)$$

$$M_3 = \sum_{i=0}^{n-1} \beta_i e_i^2 \left(\beta_i^2 e_i - 3\beta_i e_i \sum_{j=i}^{n-1} \beta_j - 3 \sum_{j=i}^{n-1} \beta_j^2 e_j + 6 \sum_{j=i}^{n-1} \beta_j e_j \sum_{k=j}^{n-1} \beta_k\right)$$

Valuation (Cont.)

where

n	the number of fixing points still available
t_i	the i-th fixing time
F_i	the forward rate at time t_i
α_I	the weight of F_i
$\beta_i = \alpha_i F_i$	
v_i	the effective volatility at time t_i where $v_i^2 t_i = \int_0^{t_i} \sigma^2(s) ds$
$e_i = \exp(v_i^2 t_i)$	

Valuation (Cont.)

- The sifted lognormal parameters are

$$y_1 = \frac{M_2 - M_1^2}{z - (M_2 - M_1^2)/z}$$

$$y_{11} = M_2 - M_1^2 + y_1^2$$

$$\delta = M_1 - y_1$$

$$z = \left(\frac{\mu_3 + \sqrt{\mu_3^2 + 4\mu_2^3}}{2}\right)^{\frac{1}{3}}$$

$$\mu_3 = M_3 - 3M_1(M_2 - M_1^2) - M_1^3$$

- By assuming that the average asset price is lognormal, you can use Black's model to price an Asian option.

Valuation (Cont.)

- The present value of an Asian call option is given by

$$PV_C = (y_1 N(d_1) - (K - \psi A - \delta) N(d_2)) D$$

$$d_1 = \frac{ln\left(\frac{\sqrt{y_{11}}}{K - \psi A - \delta}\right)}{\sqrt{\ln(y_{11}/y_1^2)}}$$

$$d_2 = d_1 - \sqrt{\ln(\frac{y_{11}}{y_1^2})}$$

where

$D = D(0, T)$ the discount factor

Valuation (Cont.)

$D = D(0, T)$	the discount factor
N	the cumulative standard normal distribution function
T	the maturity date
ψ	the sum of weights corresponding to spent fixing periods
A	the spent average
K	the strike

- The present value of an Asian put option is given by

$$PV_p = \big((K - \psi A - \delta)N(-d_2) - y_1 N(-d_1)\big)D$$

Notes

- First, you need to construct interest rate zero curves for both base and quote currencies.
- The curve construction in FX world is different from the one in interest rate world.
- Second, you need to construct an arbitrage-free volatility surface. FinPricing is using Vanna Volga model to construct FX volatility surface.
- After that, you can use the formulas to calculate the price and risk sensitivities.

Currency Asian

Example

Name	Value	Fixing Date	Fixing Value
Reporting Currency	USD	1/6/2017	1.2283
Average Type	Rate	1/13/2017	1.2219
Call Put	Put	1/20/2017	1.236
Buy Sell	Sell	1/27/2017	1.2554
Delivery Type	Cash	2/3/2017	1.2481
Base Currency	GBP	2/10/2017	1.2486
Base Currency Notional	9300000	2/17/2017	1.24635
Underlying Currency	USD	2/24/2017	1.24575
Underlying Currency Notional	11490150	3/3/2017	1.2298
Trade Date	1/5/2017		
Maturity Date	3/31/2017		
Settlement Date	4/4/2017		
Strike and Spot Quotation	USD/GBP		
Strike Value	1.2355		
Instrument	GBP/USD		
Fixing Date	3/3/2017		

Reference:

https://finpricing.com/lib/EqLookback.html